



JOINT UNIVERSITIES PRELIMINARY EXAMINATIONS BOARD

JUNE 2020 EXAMINATIONS

JUPEB/015

MATHEMATICS

SCI – J154

Time Allowed: 3 Hours

SECTION A: MULTIPLE CHOICE QUESTIONS

Answer all questions in this section.

Use the OMR answer sheet provided to answer the questions. Follow the instructions on the OMR sheet.

SECTION B: ESSAY QUESTIONS

Answer FOUR questions; ONE question from each course.

SECTION A: MULTIPLE CHOICE QUESTIONS.

Answer all questions in this section.

- For what values of k will the equation $y^2 - (k - 2)y + 1 = -2k$ have real equal roots?
 - 0, 12
 - 3, 4
 - 0, 3
 - 3, 4

- Which of the following is correct?
 - $270^\circ = 2\pi$ radians
 - $360^\circ = 3\pi$ radians
 - $90^\circ = \pi$ radians
 - $720^\circ = 4\pi$ radians

- Given that A and B are subsets of a universal set U , which of the following is true?
 - $(A \cup B)^c = A^c \cup B^c$
 - $(A \cap B)^c = A^c \cup B^c$
 - $(A \cup B)^c = (A \cup B) \cap (B \cup A)$
 - $(A \cap B)^c = A^c \cap B^c$

- Express the complex number $2 + 3i$ in polar form.
 - $3.6(\cos 28.1^\circ + i \sin 28.1^\circ)$
 - $3.6(\cos 56.31^\circ + i \sin 56.3^\circ)$
 - $4.8(\cos 36.2^\circ + i \sin 36.2^\circ)$
 - $4.8(\cos 56.3^\circ + i \sin 56.3^\circ)$

- Let $f : x \rightarrow \frac{x+1}{x-2}$ be a function defined on the set of real numbers excluding 2.

Determine the inverse of the function.

- A. $\frac{2x-1}{x-1}$
- B. $\frac{2x+1}{x-2}$
- C. $\frac{2x+1}{x-1}$
- D. $\frac{x-1}{2x+1}$
6. Obtain the range of values for which $|2x - 3| < 5$.
- A. $-1 < x < 4$
- B. $1 < x < -4$
- C. $-4 < x < 1$
- D. $1 > x > 4$
7. Find the constant term in the binomial expansion of $\left(2x + \frac{1}{x}\right)^8$.
- A. 820
- B. 1120
- C. 1210
- D. 4220
8. The end coordinates of the diameter of a circle are (3, 2) and (9, 6). Formulate the equation of the circle.
- A. $x^2 + y^2 - 6x - 4y + 39 = 0$
- B. $x^2 + y^2 + 6x + 4y - 36 = 0$
- C. $x^2 + y^2 - 12x - 8y + 39 = 0$
- D. $x^2 + y^2 + 12x + 8y - 39 = 0$
9. Simplify $\log_7 8 \times \log_6 7 \div \log_6 5 \times \log_8 5$
- A. $\log_{10} 1680$
- B. 1

- C. $\log_{48} 56$
D. 2
10. Obtain the equation of a straight line joining points $(-2, 3)$ and $(1, 4)$.
A. $3y - 11x - 1 = 0$
B. $5y - 3x - 4 = 0$
C. $3y - 2x + 11 = 0$
D. $3y - x - 11 = 0$
11. Derive the solution of the equation: $\cot \theta + \tan \theta = 2 \operatorname{cosec} \theta$.
A. $\frac{\pi}{2}$
B. $\frac{\pi}{3}$
C. $\frac{\pi}{4}$
D. $\frac{\pi}{5}$
12. Formulate a polynomial whose quotient and remainder when divided by $x + 1$ are $x^2 + 3x - 2$ and -3 respectively.
A. $x^3 - x^2 + 3x - 6$
B. $2x^3 + 3x^2 - x + 4$
C. $x^3 + 4x^2 + x - 5$
D. $x^3 - x^2 + 3x - 6$
13. Two matrices are defined as: $A = \begin{bmatrix} 6 & y & 2 \\ -4 & -2 & -6 \\ 10 & 8 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 & 5 \\ 2 & -1 & 2x \\ 1 & -3 & 1 \end{bmatrix}$. If $A^T = 2B$, evaluate $(2x + y)$.
A. 12
B. 10
C. 8

D. 6

14. A binary operation \otimes is defined on the set \mathbb{R} of real numbers such that for $x \neq 0, y \neq 0 \in \mathbb{R}$,

$$x \otimes y = \frac{3x^3 + 4y}{5xy}.$$

Evaluate $(1 \otimes 3)$.

- A. -1
B. $-3/5$
C. $-2/3$
D. 1
15. If $\lim_{x \rightarrow c} f(x) = l$ and $\lim_{x \rightarrow c} f(x) = m$, then
- A. $l < m$.
B. $l > m$.
C. $l = m$.
D. $l \leq m$.

16. Write the order and degree of the differential equation $\frac{d^2y}{dx^2} + a^2x = 0$.

- A. 2, 2
B. 1, 2
C. 2, 3
D. 2, 1
17. If $f(x) = 3^{x-1}$, find $f(x+3) - f(x)$ in terms of $f(x)$.
- A. $\frac{15}{2}f(x)$
B. $\frac{13}{2}f(x)$
C. $80f(x)$
D. $27f(x)$

18. Express $\frac{d^2y}{dx^2}$ in terms of y alone in the equation $x^2 + y^2 = r^2$, where $r = \text{constant}$.

A. $\frac{y^2 - r^2}{y^3} - \frac{1}{y}$

B. $-\frac{y^3}{r^2}$

C. $\frac{r^2}{y^3}$

D. $\frac{y^3}{r^2}$

19. Determine $\frac{dy}{dx}$ where $x = a(1 + \sin \theta)$ and $y = a(1 - \cos \theta)$.

A. $\sin \theta + \cos \theta$

B. $\tan \theta$

C. $\sec^2 \theta$

D. $\tan \frac{\theta}{2}$

20. Determine the value of the integral $\int_{-2}^{-1} \frac{dx}{x}$.

A. $\log(-2)$

B. $\log \frac{1}{2}$

C. $\log(-1) - \log(-2)$

D. $\log(-1)$

21. Let $f(x) = \begin{cases} x - 1, & \text{when } x < 1 \\ x^2 + k, & \text{when } x \geq 1 \end{cases}$ find the value of k for which $f(x)$ is continuous at $x = 1$.

A. 0

B. 1

C. -1

D. 2

22. Find $\frac{dy}{dx}$ if $y = \log_e \sin x$.
- A. $-\cot x$
 - B. $\cot x$
 - C. $\tan x$
 - D. $-\tan x$
23. Solve the differential equation $\frac{dy}{dx} - xy^3 = 0$
- A. $C = \frac{x^2+y^2}{2}$
 - B. $\frac{x^2y^2}{2} = C$
 - C. $-\frac{x^2}{2} - \frac{1}{2y^2} = C$
 - D. $\frac{x^2y^2+2}{2} = C$
24. Differentiate $y = e^{2x}\cos x$ using product rule.
- A. $e^{-2x}(2\cos x - \sin x)$
 - B. $e^{2x}(2\cos x - \sin x)$
 - C. $e^{2x}(2\cos x + \sin x)$
 - D. $e^{-2x}(2\cos x + \sin x)$
25. Integrate 4^x with respect to x .
- A. $\frac{6^x}{\ln 6} + C$
 - B. $\frac{4^x}{\ln 4} + C$
 - C. $\frac{5^x}{\ln 5} + C$
 - D. $\frac{2^x}{\ln 2} + C$

26. Integrate $\frac{\cos x}{1 + \sin^2 x}$ with respect to x .
- A. $\tan^{-1}(\sin x) + C$
 - B. $\sin x \tan x + C$
 - C. $\sin^{-1}(\tan x) + C$
 - D. $\tan x - 1 + C$
27. Evaluate $\int \frac{3x^2 - 2x + 4}{x^3 - x^2 + 4x - 18} dx$.
- A. $\ln|x^3 - x^2 + 4x - 18| + C$
 - B. $\ln|x^3 - x^2 + 4x - 18|$
 - C. $3\ln x^3 - 2\ln x^2 + 4\ln x - 18x + C$
 - D. $\ln|3x^2 - 2x + 4| + C$
28. Evaluate $\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x}$.
- A. 0
 - B. 4
 - C. 1
 - D. 2
29. A vector F is said to be irrotational if
- A. $\text{div}(\text{curl } F) = 0$
 - B. $\text{div } F = 0$
 - C. $\text{curl}(\text{div } F) = 0$
 - D. $\text{curl } F = 0$
30. The basic types of friction that exist between two surfaces in contact are
- A. limiting and static friction.
 - B. limiting and sliding friction.
 - C. sliding and dynamic friction.
 - D. kinetic and dynamic friction.

31. A fire fighter 50m away from a burning building directs a stream of water from a fire hose at an angle of 30° above the horizontal. If the velocity of the stream is 40m/s, at what height will the stream of water strike the building? (Take $g = 9.8 \text{ m/s}^2$)
- A. 20 m
B. 18.7 m
C. 8.45 m
D. 1.0 m
32. A 15kg block is pushed up a 35° incline. A friction force of 110N exists between the block and the incline. What minimum force F , would be necessary to move the block up the incline at a constant speed? (Take $g = 9.8 \text{ m/s}^2$)
- A. 26 N
B. 84 N
C. 150 N
D. 194 N
33. Find r such that $\vec{u} = 8\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{v} = \hat{i} + 5\hat{j} + r\hat{k}$ are orthogonal vectors.
- A. $\frac{1}{2}$
B. 1
C. -1
D. $-\frac{1}{2}$
34. Find the moment of inertia of a solid sphere 2m in diameter and of mass 5kg about an axis passing through the center.
- A. 2 kgm^2
B. 3 kgm^2
C. 4 kgm^2
D. 5 kgm^2
35. Obtain the momentum of a body with mass 2kg and velocity 5 m/s.
- A. 9 Ns

- B. 8 Ns
C. 10 Ns
D. 11 Ns
36. The force generated from the engine of a car is found to be 300 N. Calculate the power developed when the car moves with constant speed of 10 m/s.
A. 30 W
B. 300 W
C. 3000 W
D. 30000 W
37. A lift moves with an acceleration of 2 m/s^2 . Estimate the reaction of the floor on an object of mass 50 kg placed on the floor of the lift when it moves up.
A. 600 N
B. 300 N
C. 200 N
D. 100 N
38. Given that $\vec{a} = 2\mathbf{i} + 3\mathbf{k}$, $\vec{b} = 5\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\vec{c} = \mathbf{i} + \mathbf{j}$, evaluate $(\vec{a} \cdot \vec{b})\vec{c}$.
A. $13\mathbf{i} - 13\mathbf{j}$
B. $13\mathbf{i} + 13\mathbf{j}$
C. $13\mathbf{i} + \mathbf{j}$
D. $\mathbf{i} + 13\mathbf{j}$
39. A body starts from rest and moves with uniform acceleration to attain a velocity of 15 m/s in 10 seconds. Evaluate its acceleration at that time.
A. 1.5 m/s^2
B. 1.6 m/s^2
C. 1.1 m/s^2
D. 0.5 m/s^2

40. Data representation in which the bars are joined together is called
- pictogram.
 - bictogram.
 - histogram.
 - bar chart.
41. If ${}^3C_r = 10$, what is the value of r ?
- 5
 - 4
 - 3
 - 2
42. A problem of Statistics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{1}{4}$ respectively. Determine the probability that the problem will be solved.
- $\frac{3}{32}$
 - $\frac{23}{32}$
 - $\frac{15}{32}$
 - $\frac{29}{32}$
43. Given that $Y = aX + b$ where X is a random variable and a and b are constants, then $E(Y)$ and $Var(Y)$ are
- $aE(X) + b, a^2Var(X)$
 - $aE(X), aVar(X)$
 - $aE(X) + bE(X), a^2Var(X) + bVar(X)$
 - $E(X), Var(X)$
44. If the probability that an individual suffers a bad reaction from a certain injection is 0.001, determine the probability that out of 2000 individuals exactly 3 will suffer a bad reaction.
- 0.12

- B. 0.18
C. 0.16
D. 0.08
45. Find the expected number of students admitted to study medicine and surgery in a random sample of 100 students if the probability of securing admission is 0.2.
- A. 20
B. 30
C. 40
D. 50
46. On a final examination in mathematics, the mean was 72, and the standard deviation was 15. Determine the standard scores of students receiving grade 60.
- A. -0.8
B. 1.4
C. 0
D. -0.6
47. Which of the following is not a simple probability sampling technique?
- A. Systematic
B. Stratified
C. Cluster
D. Quota
48. Obtain a linear regression equation of Y on X using the following information. $n = 5$, $\Sigma x = 25$, $\Sigma x^2 = 165$, $\Sigma y = 24$, $\Sigma y^2 = 124$ and $\Sigma xy = 144$.
- A. $0.8 + 0.6X$
B. $1.8 + 0.6X$
C. $2.6 + 0.6X$
D. $3.8 + 0.6X$

49. The following data regarding the heights (y) and the weights (x) of 5 college students are given: $\sum x = 15$; $\sum y = 10$; $\sum xy = 165$; $\sum x^2 = 325$; $\sum y^2 = 125$. Evaluate the correlation coefficient between the height and the weight.
- A. -0.005
 - B. 0.005
 - C. 0.05
 - D. -0.05
50. The sampling technique that allows the investigator to consciously or unconsciously pass personal judgment to influence the sample selection process is identified as...
- A. stratified random sampling.
 - B. systematic sampling.
 - C. non-probability sampling.
 - D. cluster sampling.

SECTION B: ESSAY QUESTIONS

Answer FOUR Questions; ONE Question from each Course.

MAT 001: ADVANCED PURE MATHEMATICS

1. (a) i. Find the determinant of $\begin{bmatrix} 2x & 1 \\ y & 2 \end{bmatrix}$ given that $2x + y = 5$ and $x - 2y = -5$.
- ii. Given that A and B are acute angles and $\sin A = \frac{2}{5}$ and $\cos B = \frac{7}{25}$.
Find without using calculator, the value of $\cos(A + B)$. **[5 marks]**
- (b) In a secondary school, 80 students played Hockey or Football. The number that played Football is 5 more than twice the number that played Hockey. If 15 students played both games and every student in the school plays at least one game. Find:
- the number of students that play football;
 - the number of students that play football but not Hockey;
 - the number of students that play Hockey but not Football. **[5 marks]**
- (c) i. Obtain the binomial expansion of $(x + 3y)^6$.
- ii. Using the expansion obtained in (i) above, evaluate $(1.03)^6$, correct to 4 significant figures. **[5 marks]**
- [TOTAL = 15 marks]**
2. (a) i. Express $(2 + 3i)^2$ in the form $a + bi$. Hence, evaluate $\frac{2b-3a}{4ab}$
- ii. Given that a complex number $Z = 2 + i$, express $1 + Z$ on an Argand diagram. **[5 marks]**
- (b) If $(x + 1)$ is a factor of the polynomial $P(x) = x^3 + kx^2 + x + 6$,
- Compute the value of k .
 - Hence, factorize the $P(x)$ completely. **[5 marks]**
- (c) Calculate the length of tangent to the circle $x^2 + y^2 + 6x - 4y + 12 = 0$ from an external point $(3, 5)$. **[5 marks]**
- [TOTAL = 15 marks]**

MAT 002: CALCULUS

3. (a) Evaluate the following:

i. $\lim_{x \rightarrow 1} \frac{\sqrt{5x-4} - \sqrt{x}}{x-1}$,

ii. $\lim_{x \rightarrow 2} \frac{x^2+2x+1}{x^2+3x+2}$. [5 marks]

(b) Evaluate $\int \sqrt{a^2 - x^2} dx$. [5 marks]

(c) Given $I_n = \int \tan^n x dx$, show that $I_n = \frac{\tan^{n-1}x}{n-1} - I_{n-2}$, hence obtain $\int \tan^5 x dx$. [5 marks]

[TOTAL = 15 marks]

4. (a) i. Solve the differential equation $(y^2 + 2)(2x + 1) \frac{dy}{dx} = 1$, $y(0) = 1$.

ii. Expand $f(x) = \sin^2 x$ as a Maclaurin's series of ascending powers of x in the first three terms. [6 marks]

(b) 60m of fencing is to be used to form three side of a rectangular enclosure, the fourth side being an existing wall. Compute:

i. the length of the rectangular enclosure in terms of the breadth.
ii. the maximum possible area of the enclosure. [5 marks]

(c) Find the volume generated when the plane figure bounded by $y = 2\cos 2x$, the x -axis and ordinates at $x = 0$ and $x = \frac{\pi}{4}$, rotates about the x -axis through a complete revolution. [4 marks]

[TOTAL = 15 marks]

MAT 003: APPLIED MATHEMATICS

5. (a) Given $\varphi(t) = e^t$; $u(t) = \sin t \hat{i} + \cos t \hat{j} + 3\hat{k}$; $v(t) = t\hat{i} - 2\hat{k}$.

i. Find (b) $\frac{d}{dt}(u \cdot v)$.

ii. Evaluate $\frac{d}{dt}(u \times v)$. [5 marks]

(b) A load of mass 50kg is placed in a lift. Calculate the reaction between the floor of the lift and the load when the lift

- i. is moving at a constant speed;
 - ii. moves upwards with an acceleration of 3m/s^2 . **[5 marks]**
- (c) A body of mass 8.2kg is supported by two light inextensible strings attached to it. The other ends of the string are attached to two fixed points in a ceiling and are 10m apart. One of the strings is 6m long and the other is 8m long. Assuming that the system is in equilibrium, calculate:
- i. the angle made by each string to the horizontal.
 - ii. the tension in each string. (Take $g=10\text{m/s}^2$) **[5 marks]**
- [TOTAL = 15 marks]**

6. (a) A 4kg ball moving with a velocity of 10m/s collides with a 16kg ball moving with a velocity of 4m/s in the opposite direction. Calculate the
- i. velocity of the balls if they collide on impact;
 - ii. loss energy resulting from the impact. **[6 marks]**
- (b) If $A = 4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$, $B = 2\mathbf{i} - 10\mathbf{j} - 7\mathbf{k}$ and $C = 5\mathbf{i} + 7\mathbf{j} - 4\mathbf{k}$. Deduce the values of :
- i. $(A \times B) \cdot C$ and $A \times (B \times C)$.
 - ii. unit vectors perpendicular to A and lying in the plane of B and C . **[5 marks]**
- (c) A particle of mass 15kg is placed on a smooth plane inclined at an angle 30° to the horizontal. Use the details above to find:
- i. the acceleration of the object as it moves down the plane.
 - ii. the velocity attained after 3seconds if it moves with an initial velocity of 5m/s .
[Take $g = 10\text{m/s}^2$] **[4 marks]**
- [TOTAL = 15 marks]**

MAT 004: STATISTICS

7. (a) The weekly wages of 1000 workmen are normally distributed around a mean of $\text{N}70$ and a standard deviation of $\text{N}5$. Estimate the number of workers whose weekly wages will be:
- i. between $\text{N}69$ and $\text{N}72$.
 - ii. more than $\text{N}75$.
 - iii. less than $\text{N}63$. **[6 marks]**

- (b) The sugar content of five cups of peak milk was measured and the result in mg/cup yields 14.5, 14.2, 14.4, 14.3 and 14.6. At $\alpha = 0.05$, test $H_0: \mu = 14.0$ against $H_1: \mu \neq 14.0$.

[5 marks]

- (c) The scores of students in Economics test have an average of 8 and standard deviation of 0.6.

- Find the Z-scores for the students whose scores are [7.5, 8.5, 6.8, 9.0]
- Find the probability of selecting at random a student whose score is between 7 and 9.

[4 marks]

[TOTAL = 15 marks]

8. (a) A certain village plagued with a certain disease is classified into two groups X and Y consisting of 300 people each after a random sample. A drug sent by the United Nations Relief Agency is administered to the group X only. The result showed that in group X and Y , 250 and 200 people respectively recovered from the disease. Verify the claim that the drug helps in curing the disease at 5% level of significance. (Hint: use $\chi^2_{0.05;1} = 3.841$).

[6 marks]

- (b) Suppose X is a random variable which represents the number of heads recorded in a fair toss of 4 coins once.

- Find the probability mass function (pmf)
- Evaluate
 - $P(X = 2)$
 - $P(X \geq 1)$

[4 marks]

- (c) A bag contains nine balls two of which are red, three blue and four black. Three balls are drawn from the bag at random. What is the probability that

- the three balls are of different colours?
- the three balls are of the same colours?

[5 marks]

[TOTAL = 15 marks]

Relevant Statistical Table

Student's t- Distribution

$\alpha=$	0.25	0.1	0.05	0.025	0.01	0.005	0.0005
v=1	1.000	3.078	6.314	12.706	31.82	63.66	636.62
2	0.816	1.886	2.920	4.303	6.965	9.925	31.599
3	0.765	1.638	2.353	3.182	4.541	5.841	12.924
4	0.741	1.533	2.132	2.776	3.747	4.604	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	4.781

Standard Normal (Z) Table

cumulative Normal probabilities										
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-1.6	0.0548	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655
-1.5	0.0668	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793
-1.4	0.0808	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951
-1.3	0.0968	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131
-1.2	0.1151	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335
-0.5	0.3085	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409
-0.4	0.3446	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783
-0.3	0.3821	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168
-0.2	0.4207	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562
-0.1	0.4602	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015

Chi-Square Table

df	0.5	0.250	0.100	0.050	0.025	0.010	0.005
1	0.4549	1.323	2.706	3.841	5.024	6.635	7.879
2	1.3863	2.773	4.605	5.991	7.378	9.210	10.597
3	2.3660	4.108	6.251	7.815	9.348	11.345	12.838
4	3.3567	5.385	7.779	9.488	11.143	13.277	14.860
5	4.351	6.626	9.236	11.070	12.833	15.086	16.750